# CS 2413 001 Summer 2000 Homework #2

## Quiz to be held in class 1:20pm Friday 23 June 2000

You will need extra sheets of notebook paper to write your answers on.

1. Using the definition of big-**O** on page 53 of *OODS*[1], for each of the following functions, if an algorithm takes that many primitive operations on an input of size $n$, **what is the time complexity** of the algorithm? **Show that your answer is correct** by choosing suitable values for the constants $c$ and $n_0$. You do not need to give a formal mathematical proof (which is beyond the scope of this course); instead, you may obtain the values of these constants by testing or by informal reasoning. **Explain** how you got your answer.

   (a) $6\,n^2 + 9\,n + 12$

   (b) $0.015\,n\log_2 n + 22\,n + 126\log n + 329$

   (c) $17\log^2 n + 8$ Note that $\log^2 n$ is the same as $(\log n)^2$.

2. In Figure 1.1 on page 55 of *OODS*, the value of $\log 10$ is 3.322, which is actually $\log_2 10$ (log to the base 2 of 10). **Does it matter** whether we use $\log_2$, $\log_{10}$ or $\log_a$ for some real number $a > 1$? **Explain.** (Hint: what is the relationship between $\log_p n$ and $\log_q n$?)

3. For each of the following problems, **is it possible to write a procedure** that will solve the problem and **that meets the definition of an algorithm** (*OODS*, page 47)? You don't need to write the algorithm, but you must **explain your answer**.

   (a) Determine whether two strings are the reverse of each other (i.e., the last character of one is the first of the other, etc).

   (b) Determine whether a given integer $x$ is a prime number (only factors are 1 and itself).

   (c) Determine whether the countries on a map can be colored with only four colors, so that no two adjacent countries have the same color.

   (d) Consider this formula: $x^n + y^n = z^n$
   Is $n = 2$ the largest integer value for $n$ for which there exist positive integers $x$, $y$ and $z$ such that the formula has at least one solution?

4. Write recursive functions in C++ to solve the following recurrence relations.

   (a) Ackerman's function:

$$\mathrm{A}(m,n) = \begin{cases} n+1, & \text{if } m = 0 \\ \mathrm{A}(m-1,1), & \text{if } n = 0 \\ \mathrm{A}(m-1,\mathrm{A}(m,n-1)), & \text{otherwise} \end{cases}$$

   (b) Greatest Common Divisor:

$$\mathrm{GCD}(m,n) = \begin{cases} m, & \text{if } n = 0 \\ n, & \text{if } m \bmod n = 0 \\ \mathrm{GCD}(n, m \bmod n), & \text{otherwise} \end{cases}$$

5. Draw the call tree (*OODS*, page 62) for the recursive algorithms from the previous question:

   (a) Ackerman's function for $m = 5$ and $n = 3$

   (b) Greatest Common Divisor for $m = 30$ and $n = 75$

[1] S. Rhadakrishnan, L. Wise & C. N. Sekharan, *Object-Oriented Data Structures Featuring C++*, 1999

6. Write a constructor for `ArrayClass` with the signature

```
ArrayClass (int arrayLength, int listLength, Object* list)
```

that constructs an instance of `ArrayClass` with `arrayLength` elements, such that:

- if `arrayLength` $\geq$ `listLength`, then all of the elements of `list` are copied into the first `listLength` elements of the `ArrayClass` instance;
- otherwise, the first `arrayLength` elements of `list` are copied into the `ArrayClass` instance.

7. Write a function that takes an instance of `ArrayClass` and returns another instance of `ArrayClass` that has the same elements, except that repeats of the same value are eliminated; for example, applying the function to an array containing 4,12,64,12,92,12,53,92,4 will return an array containing 4,12,64,92,53. (Hint: use a bucket array.)